Linguistic Markers to Improve the Assessment of students in Mathematics: An Exploratory Study

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Pépite project

- **Objective**
  - To provide teachers with software support to assess their students in elementary algebra
- **Bases**
  - A research in Didactics of mathematics
  - Model of algebraic competence (Grugeon 95)
    - Errors
    - Main students' conceptions and misconceptions
- **Iterative methodology**
  - Design & implementation of a first prototype (Jean & al 1999)
  - Experimentation of this prototype with teachers (Delozanne & al 2003)

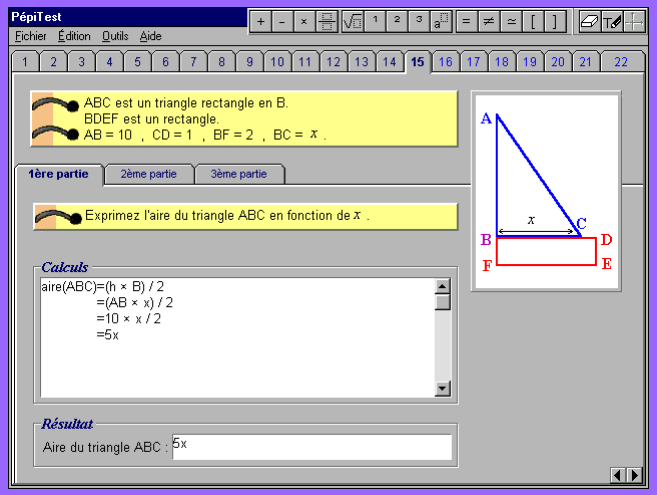
Pépite software

End-users:

Students

Teachers
Students ?

PÉPITEST



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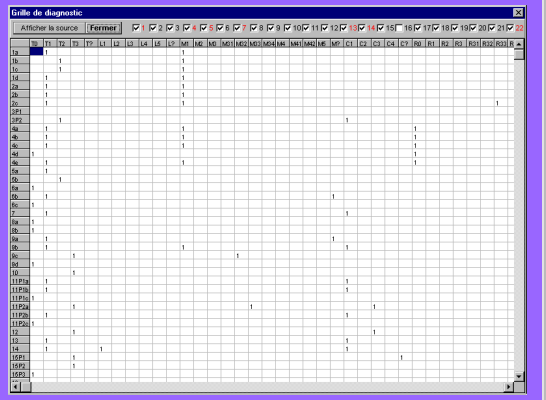

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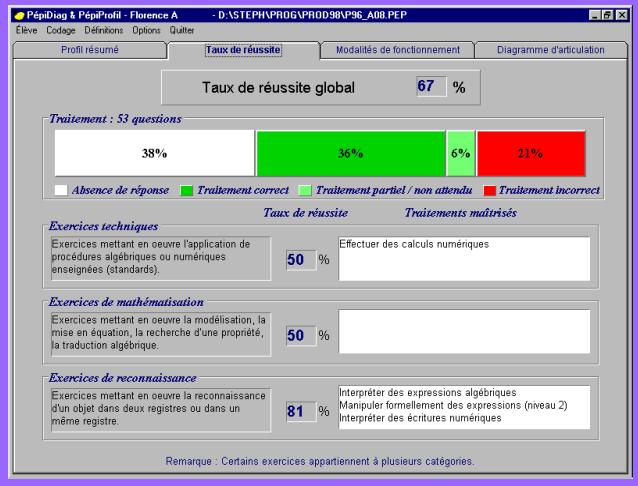

Coding of the data

Coding correction

Transversal analysis

Example of student's justifications

PépiTest

File Edit Tools Help

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22

Say if the following properties are true for each value of a.
Justify your answer.

$a^3a^2 = a^5$	(•) true ( ) false	3+2=5
$a^2 = 2a$	( ) true (•) false	Il faut multiplier les a au carré (a squared have to be multiplied)
$2a^2 = (2a)^2$	( ) true (•) false	car on ne peut pas enlever les parrenthèses (because removing parenthesis is not allowed)

Pepite1's analysis for $a^3\ a^2 = a^5$

Proof by…	Type of justifications	Representation modes	Examples of students' justifications
Algebra (correct)	To give a correct rule or definition	Algebraic	$a^2\ a^3 = a^{2+3}$ $a^n\ a^p = a^{n\times p}$
Numerical example (incorrect)	To try with one or several numbers	Numerical	$3^3 \times 3^2 = 3^5$
Explanation (incorrect)	To give explanations	Mathural language	"It's true because both exponents are added " "In multiplications with powers, exponents are added" "The product of two identical numbers with different exponents is t his same number but with both exponents added, thus a to the power 2+3"
School authority (incorrect)	To rely on authority	Mathural language	"We must never multiply exponents" "It's a fundamental law"

Research objectives

- **Hypothesis :**
  - Formulations in mathural language might demonstrate an early level of comprehension
  - A linguistic study might give insights to improve the classification of students' justifications
- **Objective**
  - To connect linguistic structures used by students with their level of development in algebraic thinking
- **Related work**
  - Aleven, Koedinger & al 2003
  - Rosé & al 2003, Graesser & al 2003

Methodology

- Step 1 : a linguistic point of view
  - An empirical analysis of a sample of data
- Step 2 : a linguistic and didactical point of view
  - A first categorization of students' justifications
- Step 3 : a workshop with researchers and teachers
  - A review of this categorization
- Step 4 : a final categorization by researchers
  - Presented here
- Step 5 : a validation of the categorization

Sample coding (by researchers)

- 168 students (aged 15-16)
  - From French secondary schools
- Focus on 52 students
  - At least one “mathural” justification (ex. 2)
- Students’ performance
  - Global : 2 groups
    - Group 1 (24) : 3 good choices for the 3 questions
    - Group 2 (28) : at least 1 wrong choice
  - Local : 4 categories of answers (CC, CP, CI, II)
    - Choice : Correct, Incorrect
    - Justification : Correct, Partial, Incorrect

Sample coding (cont.)

- For each question
  - Equality characteristics from a mathematical point of view
  - Description of the linguistic structures used by students in each performance category (CC, CP, CI, II)
  - Definition of a typology of students' justifications based on 4 discursive modes

    Argumentative, Descriptive, Explanatory, Legal
  - We hypothesized that

    These discursive modes were linked with different levels of developments in algebraic thinking

    Conceptual, Contextual, School Authority

A priori correlation

Correctness	Discursive modes	Level of development in algebraic thinking	
CC	*Argumentative :* Students use connections between their arguments to articulate their justifications (consequence, restriction, opposition, coordination)	*Conceptual :* Students handle concepts	ALGEBRA THINKKING
CC, CP	*Descriptive :* Students describe some elements f rom the context set by the given equality .	*Contextual :* Students select some elements that make sense in the context	
CP, CI, II	*Explanatory :* Students require causality often with wrong arguments.	*School authority :* Students apply or mention formal rules or malrules without mentioning a context of validity	
	Legal : Students base their justification on legal or authoritative arguments		

Classification for $a^3\ a^2 = a^5$

Code	Discursive mode	Algebraic thinking	Example	Linguistic features	Number
CC	Argumentative (consequence, restriction)	Conceptual	The product of 2 identical numbers with different exponents is this same number but with exponents both added, thus a to the power 2+3	Complex sentence but, thus	3 (on 9 CC) 3 Gr1
CC	Descriptive	Contextual	When you multiply the same numbers with powers, you add the power and the number remains unchanged	Complex sentence when, at the time of, in	5 (on 9 CC) 1 Gr1 4 Gr2
CP			In multiplications with powers, exponents are added		15 (on 26 CP) 12 Gr1, 3 Gr2
CP	Explanatory	School authority	Because it is necessary to add the powers	Short sentence Because, it's true because, it is necessary	6 (on 26 CP) 6 Gr2
II	Legal	School authority	We are not allowed to add the powers but we have to multiply them	Short sentence it is necessary, you have to you are not allowed	4 (on 8) 4 Gr2
Not classified			1 CC (on 9), 3 CP (on 26), 5 CI (on 5), 4 II (on 8), 4 non justified		

Classification for $a^2 = 2a$

Code	Discursive modes	Algebraic thinking	Example	Linguistics Features	Number
CC	Argumentation opposition	Conceptual	Squared a means a×a while 2a means a×2	while, whereas, and not	11 (on 22 CC) 9 Grt1, 2 Gr2
CC	Argumentation coordination	Conceptual	Because the first results in a times a and the second results in twice a	and, thus	9 (on 22 CC) 5 Gr1, 4 Gr2
CP	Descriptive	Contextual	It results in a×a, It is « a+a » who is equal to 2a	it is, it result s in, it is equal to a	5 (on 11 CP) 3 Gr1, 2 Gr2
II	Explanatory	School authority	It is true because the squared letter a results in 2a (a×a = 2a).	because, it is true because	6 (on 10 II) 6 Gr2
Not classified			2 CC (on 22), 6 CP (on 11), 3 CI (on 3), 4 II (on 10), 6 no justifications		

Classification for $2a^2 = (2a)^2$

Code	Discursive modes	Algebraic thinking	Example	Linguistic Features	Number
CC	Argumentation opposition	Conceptual	In the first part of the equation, only a is squared while in the second part, the product of 2a is squared	while, whereas, and not	14 (on 19 CC) 12 Gr1 2 Gr2
CC	Argumentation coordination	Conceptual	Because $2a^2$, it is a that is squared. And $(2a)^2$, it is 2a that is squared.	and, because	5 (on 19 CC) 3 Gr1 2 Gr2
CP	Descriptive, restriction	Contextual	As there is no parenthesis, only the value a is to be multiplied by itself	only	4 (on 8 CP) 2 Gr1 2 Gr2
II	Explanatory	School authority	Because you multiply from left to right	because	2 (on 5 II) 2 Gr2
II	Legal	School authority	It is allowed to put parenthesis to a digit	It is allowed to, you can	2 (on 5 II) 2 Gr2
Not classified			2 CP (on 8), 3 CI (on 3), 1 II (on 5), 16 not justified		

First Validation

- Students from
  - Gr. 1 : Conceptual and contextual level
  - Gr. 2 : (mostly) Contextual and School authority level
- With some students,
  - We began testing the correlation between
    - Level of development in algebraic thinking (assigned as described here)
    - Cognitive profile in algebra set by Pépite based on the whole test
  - the distinction between school authority/contextual/conceptual was relevant
    - Students at school authority level often invoke malrules in calculations
    - Students at conceptual level obtain good results for the whole test
    - Students at contextual level are not so predictable
      - Descriptive mode seems sensitive to the given equality

New hypothesis

- $a^3\ a^2 = a^5$
  - True, similar to a rule studied in math lessons
  - Good choice : Few argumentation ; more description
  - Bad choice : More legal
- $a^2 = 2a$
  - False, not in relation with math lessons
  - Good choice : more argumentation (opposition and coordination)
  - Bad choice : few legal, more explanatory
- $2a^2 = (2a)^2$
  - False, parenthesis emphasized in math lessons
  - Good choice : more argumentation, few description
  - Bad choice : some legal
- New hypothesis
  - The discourse mode might be influenced by the mathematical features of the equality
    - True or false statement
    - Complexity of the expressions
    - Invariant or variant elements from one side to the other
    - Proximity with math lessons (parenthesis, rules etc.)

Results & perspectives

- **A categorization of students' justifications**
  - Based on hypothesized links between
    - **Performance, Discursive modes and Level of Development in algebraic thinking**
  - Applied to a sample of 58 students
- **Future validation**
  - To systematically triangulate
    - **Performance**
      - **Correctness of the justifications**
    - **Level of development in algebraic thinking**
      - **Classification based on linguistic markers**
    - **Students' competence**
      - **Assessed by PépiTest with the whole test**
  - **To test the categorization on other sample**
    - **Same task**
    - **Modulating the mathematical features of the equality**

Web sites

Software download and research documents

http://pepite.univ-lemans.fr

Documents for teachers (in French)

http://maths.creteil.iufm

(formation continue, la compétence algébrique du collège au lycée)

Types of justification (N= 176)

Grade	Q.	ALG.	NUM.	M. L.	NO
3° (gr 9) N= 96	Q1	17%	3%	30%	51%
	Q2	16%	-	26%	58%
	Q3	10%	-	30%	60%
	Total	14%	1%	28%	57%
2° (gr 10) N= 80	Q1	26%	9%	44%	20%
	Q2	44%	-	35%	21%
	Q3	30%	-	33%	37%
	Total	33%	3%	38%	26%